In a nutshell: Gaussian elimination with partial pivoting

Given a system of n linear equations in n unknowns $A\mathbf{u} = \mathbf{v}$, our goal is to find $\mathbf{u}$ if it is unique, or if there are zero or infinitely many solutions.

1. Set up the augmented matrix $A_{\text{aug}} \leftarrow \left(A \mid \mathbf{v} \right)$ and denote the entries of this matrix $a_{i,j}$.
2. Set $i, j \leftarrow 1$.
    a. If $j = N + 1$, we are done.
    b. Find the largest entry in absolute value for the values $a_{k,j}$ where $k = i + 1, \ldots, n$. If there are multiple entries that are largest in absolute value, just pick one. Let m be the entry that was chosen.
        (i) If all entries are zero, there are either zero or infinitely many solutions, so increment j and return to Step 2*a*.
    c. Swap Rows i and m.
    d. Set $k \leftarrow i + 1$,
        (i) If $k = N + 1$, increment i and j and return to Step 2*a*.
        (ii) Add $-\frac{a_{k,j}}{a_{i,j}}$ each non-zero entry of Row i onto the corresponding entry of Row k.
        (iii) Increment k and return to Step 2*d*(*i*).
3. The augmented matrix is now in row-echelon form:
    a. If we did not flag that there may be either zero or infinitely many solutions, then there is one unique solution, and we can find that solution using backward substitution,
    b. otherwise, if there are rows where the last entry is non-zero and all other entries are zero, then there are no solutions,
    c. otherwise, there are infinitely many solutions, and each column that does not have a leading non-zero entry corresponds to a free variable.